# Heat Generation Associated with Drawing of Poly(ethylene Terephthalate)

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### **Synopsis**

The heat associated with drawing of poly(ethylene terephthalate) fiber is estimated on the basis of equations developed in the literature on films undergoing deformation by neck propagation. The deformation process is divided into two steps: neck propagation or drawing to the natural draw ratio and uniform deformation accompanied by crystallization. The results show that heat loss is negligible during deformation by necking and the temperature rise is estimated to be about 60K in yarns with a spun birefringence of 0.011. The heat released in step 2 is sufficient to raise the fiber temperature about 55K under adiabatic conditions, but under conditions of free air convection, the temperature rise is estimated to be only about 5–10K.

## INTRODUCTION

That heat is generated during the deformation of polymers has been recognized for many years. Considerable attention has been given to the role of heat generation in necking: in particular, Haward has made several contributions to the field, the most recent of which contains a brief summary of the literature.<sup>1</sup> Early work by Marshall and Thompson,<sup>2</sup> summarized by Ziabicki,<sup>3</sup> shows that the temperature rise on drawing PET can be calculated on the basis of an energy balance equation on which temperature-dependent viscoelastic functions are substituted. Most of the previous experimental work was conducted on films, primarily because of the ease of study. Direct and accurate measurement of the temperature of a fiber under nonequilibrium steady state conditions is a difficult proposition. Contact devices can remove a substantial amount of heat, can produce additional heat through friction if the fiber is being processed, and may lack the necessary response time. Temperature-sensitive crystal phosphors suffer from the problem of adhesion under drawing conditions. Noncontact instruments, such as infrared cameras and microscopes, can be difficult to focus or align and, most importantly, can be difficult to calibrate. Using data derived from measurements on films, however, some useful calculations can be made on the heat rise of a fiber that undergoes simultaneous nonuniform and uniform deformation plus crystallization. The drawing behavior of poly(ethylene terephthalate) is modeled by the calculations in this note. The calculation that follows is designed to be a "first approximation." Consequently, the reader may disagree with some of the details, but the numbers that result are useful estimates of the temperatures generated during the drawing of PET.

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#### THEORETICAL

In the paper by Maher et al.,<sup>1</sup> the authors generate an equation for the temperature rise associated with polymer necking. They focused an infrared camera (Thermovision) on a variety of polymeric films and the results, expressed mathematically, show

$$\frac{1}{\Delta T} = \frac{-hA}{Lg\alpha V_c} + \frac{ab\rho C_p}{(D-1)Lg\alpha}$$
(1)

where  $\Delta T$  is the temperature rise (°C),  $h = \text{heat loss/unit area unit time for each degree above ambient, <math>A = \text{deformation surface area (m<sup>2</sup>)}$ , L = draw tension (kg), g = 9.8 N/kg,  $\alpha = \text{efficiency } \sim 1$ ,  $V_c = \text{neck velocity (m/s)}$ , a,b = specimen width, thickness (m),  $\rho = \text{density (kg/m<sup>3</sup>)}$ ,  $C_{\rho} = \text{heat capacity (J/kg-K)}$ , and D = draw ratio.

The first term in this expression is a heat loss term; the second a heat generation term due to plastic deformation. This expression can be conveniently rewritten for the case of drawing PET fibers:

$$\frac{1}{\Delta T} = \frac{-hA}{Lg\alpha V_c} + \frac{\pi r_i^2 \rho C_p}{(D-1)Lg\alpha} + \frac{C_p}{\chi \cdot \Delta H_c}$$
(2)

where  $r_i$  = initial fiber radius,  $\Delta H_c$  = heat released during crystallization per gram of crystal, and  $\chi$  = crystallinity. The first two terms here are as in eq. (1). The third term is associated with the latent heat of stress-induced crystallization.

In the section that follows drawing is envisioned as a two-step process. Step 1 entails drawing to the "natural draw ratio." The mode of deformation is necking, and there is no crystallization. The nominal draw temperature is about 293K. Step 2 entails uniform deformation/drawing and is accompanied by crystallization. The nominal draw temperature is about 353K. Commercial draw schemes may, in fact, utilize two or more draw zones similar to those outlined here.

### CALCULATIONS

To define the neck geometry, samples were removed from the draw zone and examined using a polarized light microscope. Drawing was conducted between a feed roll and a draw roll on a 33-filament yarn. PET yarn with a filament denier of 16, a spun birefringence of 0.011, and drawn at 293K has<sup>1</sup> a characteristic natural draw ratio of 2.4 and a neck geometry as sketched:

$$\rightarrow \leftarrow 19 \,\mu\text{m}$$

$$\uparrow \qquad \uparrow$$

$$40 \,\mu\text{m} \qquad 27 \,\mu\text{m}$$

$$\downarrow \qquad \downarrow$$

The following values have been used for calculations relating to step 1: filament is denier = 16 init, 6.7 final; Draw force  $L = 0.55 \text{ g/d} = 8.8 \times 10^{-3} \text{ kg}$ ; neck velocity = output roll speed,  $V_{(\text{neck})} = 0.2 \text{ m/s}$ ; initial cross-sectional area  $\pi r_i^2 = 1.32 \times 10^{-9} \text{ m}^2$ ; draw ratio D = 2.4; deformation area =  $2 \times 10^{-9} \text{ m}^2$ ; heat capacity at 293 K,  $C_p = 4.2 [(0.247 + 9.9 (10^4) T (^{\circ}\text{C})] \cdot 1.12 \text{ kg/kJK}^4$ ; density  $\rho = 1340 \text{ kg/m}^3$ ;

heat transfer coefficient  $h = 1.17 \times 10^8 \text{ J/m}^2 \cdot \text{s-K}$  (see appendix). Substitution into eq. (2) gives

$$\frac{1}{\Delta T} = -\frac{hA}{LgV_c} + \frac{\pi r_i^2 \rho C_p}{L(D-1)g}$$
$$\frac{1}{\Delta T} = -6.4 \times 10^{-6} + 1.543 \times 10^{-2}$$

but since  $6.4 \times 10^{-6} \ll 1.64 \times 10^{-2}$ ,  $\Delta T \propto 61$ K.

One notes immediately that  $\pi r_i^2$  and L scale together (ignoring changes in L with m) so the calculated value of  $\Delta T$  is independent of filament denier. Thus, under ideal conditions (i. e., dry, neck located in air, etc.), the temperature rise is approximately 60K when the draw ratio is 2.4. For fibers of lower initial orientation, the natural draw ratio would be larger and, consequently,  $\Delta T$  would be larger. Maher et al.<sup>1</sup> report values of only about 60K, Yegerow et al.<sup>5</sup> report values in excess of 200K, and Marshall and Thompson<sup>2</sup> report a value of about 58K. Note that, since the loss term is negligible, the calculation is identical to that if one had assumed adiabatic conditions; that is, air flow around the fiber is inconsequential. Hence, the calculation holds for a yarn, tow, or single filament.

The following values have been used for calculations relating to step 2: filament denier = initial 6.7, final 3.4; draw force  $L = 1.5 \times 10^{-3}$  kg; velocity = output roll speed V = 0.3 m/s; cross-sectional area,  $\pi r_i^2 = 5 \times 10^{-10}$  m<sup>2</sup>; draw ratio D = 2 deformation area  $\pi dl$  70 × 10<sup>6</sup> m<sup>2</sup> if l = 1 m; heat capacity  $C_p$  at 353 K<sup>4</sup> 1.37 kJ/kg·K; heat of fusion (kg of crystal<sup>4</sup> 13 kJ/kg; crystallinity)  $\chi = 40\%$ ; density  $\rho = 1360$  kg/m<sup>3</sup>.

Substitution into eq. (2) gives

$$\frac{1}{\Delta T} = -\frac{hA}{LgV_c} + \frac{\pi r_i^2 \rho C_p}{(D-1)Lg} + \frac{C_p}{X \cdot \Delta H_c}$$
  
= -8.7 × 10<sup>-2</sup> + 6.3 × 10<sup>-2</sup> + 2.7 × 10<sup>-2</sup>  
( $\Delta T$  = 17) ( $\Delta T$  = 36)

Thus, crystallization alone generates sufficient heat to raise the temperature 17K, deformation alone 36K, but the loss term is sufficiently large to dissipate all the heat long before the fiber has traveled one meter. In the adiabatic or "worst" case, which could arise under certain conditions, the fiber temperature could rise, 17K + 38K = 45K. One rather unusual way to achieve adiabatic conditions in this case is simply to decrease the length of the draw zone from 1 m to a few cm.

#### SUMMARY

Heat flow equations based on measurements of films and developed in the literature have been used to estimate the heat released during two-step drawing of PET fiber. The results show that a temperature rise of about 60K from 20K is produced during neck propagation drawing of weakly oriented PET ( $\Delta n = 0.011$  and draw ratio = 2.4:1). Heat losses are negligible; consequently, the calculation is identical to that in which one assumes adiabatic conditions. Hence,

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the calculation is good for a single filament, a yarn, or a tow.

Once local deformation has ceased, uniform deformation accompanied by crystallization occurs in drawing. Under adiabatic conditions, at 80°C, deformation produces a temperature rise of about 17K and crystallization about 38K. However, when the fibers are under conditions where free air convection can occur, the temperature rise is much smaller, ca. only 5–10K.

# APPENDIX: CALCULATION OF HEAT TRANSFER COEFFICIENT

The heat transfer coefficient h is calculated assuming that the fiber is stationary and horizontal in air which cools the fiber by convection. Since the fiber in reality is moving along the fiber axis, the calculation, although being a good approximation, underestimates the cooling rate slightly. Also, the calculation is for a 23  $\mu$ m diameter cylinder (5 dpf) that is maintained at 425K in 353K air. From Bird, Steward, and Lightfoot,<sup>6</sup>

$$h = \operatorname{Nu} k/2r$$
, where  $k = \text{thermal conductivity} = 10^4 \operatorname{cal/cm} \cdot \mathrm{s} \cdot \mathrm{K}^7$  (7)

and

$$Nu = 5.25 (GrPr)^{1/4}$$

in which

$$\operatorname{GrPr} = \frac{(2r)^3 (\rho_{\operatorname{air}})^2 g(\Delta T/T)}{u_{\operatorname{air}} \cdot k_{\operatorname{air}}} \times C_{\rho_{\operatorname{air}}}$$

where  $\rho$ ,  $C_p$ , u, and k are the density, heat capacity, viscosity, and thermal conductivity of air, respectively, and g is the gravitational acceleration.

Substituting (5 dpf fiber,  $\Delta T = 60$ K, T = 350K):

$$GrPr = \frac{(9.2 \times 10^{-5})^3 (0.723)^2 (4.17 \times 10^8)}{(0.046) (0.0152)} \left(\frac{60}{350}\right) (0.241)$$
  
= 9.4 × 10<sup>-6</sup>

Thus, Nu =  $0.525 (0.09 \times 10^{-4})^{1/4} = 0.029$  and

$$h = (0.029) \left( \frac{10^{-4}}{23} \times 10^{-4} \right) \approx 1.3 \times 10^{-3} \text{ cal/cm}^2 \cdot \text{s} \cdot \text{K}$$
$$= 55 \text{ J/m}^2 \text{sec} \cdot \text{K}, \quad 350 \text{K}, \quad \Delta T = 60 \text{K}, \quad 5 \text{ dpf}$$

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